

# The Generation of Domestic Electricity Load Profiles through Markov Chain Modelling

**Fintan J. McLoughlin<sup>1</sup>, Aidan Duffy<sup>1</sup> and Michael Conlon<sup>2</sup>**

<sup>1</sup> Department of Civil and Structural Engineering, School of Civil and Building Services, Dublin Institute of Technology, Bolton Street, Dublin 1, Ireland

<sup>2</sup> School of Electrical Engineering Systems, Dublin Institute of Technology, Kevin St, Dublin 8, Ireland

**Abstract:** Micro-generation technologies such as photovoltaics and micro-wind power are becoming increasingly popular among homeowners, mainly a result of policy support mechanisms helping to improve cost competitiveness as compared to traditional fossil fuel generation. National government strategies to reduce electricity demand generated from fossil fuels and to meet European Union 20/20 targets is driving this change. However, the real performance of these technologies in a domestic setting is not often known as high time resolution models for domestic electricity load profiles are not readily available. As a result, projections in terms of reducing electricity demand and financial paybacks for these micro-generation technologies are not always realistic.

Domestic electricity load profiles are often highly stochastic, influenced by many different independent variables such as environmental, dwelling and occupant characteristics that shape individual customer's load across a single day. This paper presents a stochastic method for generating electricity load profiles based on the application of a Markov chain process. Electricity consumption was recorded at half hourly intervals over a six month period for five individual Irish dwelling types and used to generate synthetic electricity load profiles. The purpose of this paper is to determine whether Markov chain modelling is an effective way of re-generating electricity load profiles for domestic dwellings and identify shortcomings with this particular technique. The results show that the magnitude component of the load profile can be reproduced effectively whilst the temporal distribution needs to be addressed further.

**Keywords:** Markov chain, electricity

## 1. Introduction

Domestic electricity use in most European countries accounts for a major proportion of overall demand. In Ireland, 32% of final electricity was consumed in the residential sector in 2008 (SEAI, 2009). This is the second largest electricity consuming sector in the economy, exceeded only by commercial and public services sectors. The EU has set stringent targets for 2020 based on a 2005 emissions baseline: a reduction of 21% in greenhouse gas emissions for the emission trading sector across the EU-27 countries and a 10% reduction for the non-trading sector across the EU. The 10% reduction across the EU-27 countries for the non-trading sector is broken up collectively for the different member states. Ireland has been assigned a target of 20% reduction in greenhouse gas emissions by 2020.

In order to effectively respond to the EU 20/20 targets, national governments will need to accurately assess the cost and emissions effects of any energy policy decisions up to 2020. In Ireland, the National Energy Efficiency Action Plan published in 2009 makes recommendations to fully investigate the role of micro-generation, such as photovoltaics and micro-wind turbines, as an alternative to traditional power generation (DCENR, 2009).

Support mechanisms for micro-generation exist across the EU to encourage the up-take of technologies in an attempt to make them more cost competitive with conventional generation. In Ireland, the level of support for micro-generation is quite small compared to other European countries like Spain and Germany where a Renewable Energy Feed in Tariff (REFIT) price of 34cent/kWh and 39cent/kWh respectively is offered for micro-generation installations (EPIA, 2010). In February 2009, the Minister for Communications and Natural Resources offered 19cent/kWh to support micro-generation but only applies to the first 4000 installations over the next three years (DCENR, 2009).

Photovoltaics and micro-wind are highly site-specific technologies. Depending upon the available resources at a particular site, energy yield will vary considerably. Furthermore, depending on site demand characteristics and the REFIT price, payback periods and greenhouse gas marginal abatement costs will vary. Manufacturers and retailers usually supply the customer with payback periods for their products based on local environmental conditions and electricity price and support mechanisms. These calculations are usually based on an average load profile, usually daily or monthly, for a typical dwelling type. However, the actual load profile for a particular dwelling rarely resembles the average, with large fluctuations between peaks and troughs throughout the course of a day.

Historically, electricity metering at a domestic level has been carried out at a low time resolution, usually on a monthly or bi-monthly basis. However, with improvements in technology, time of use metering is now becoming more prevalent, with large energy utilities throughout Europe trialling the technology. In this paper the first stage of a Markov chain model is presented to generate high time resolution load profiles for five individual dwelling types in Ireland. Markov chain is a type of Monte Carlo analysis where probability distributions determine the likelihood of a dwelling consuming a particular load. It is suited to modelling stochastic processes such as that relating to the generation of domestic electricity load profiles.

## 2. Methodology

Domestic electricity load profiles are usually cyclical with typically a morning and evening peak and a small base load over the night time period. The load profile is shaped by switching on/off individual electrical appliances which is influenced by various environmental, dwelling and occupant characteristics. Although some appliances are cyclical, other appliances may appear to be switched on and off at random. Firth et al. (2008) looked at groups of electrical appliances (continuous and standby, cold appliances and active appliances) and examined periods of the day with which they are likely to be switched on. Continuous and standby appliances tend to form a base load with the switching in and out of cold appliances across a 24 hour period. Electricity consumption from active appliances such as kettles and electric showers are more random and typically have high power requirements.

Wood and Newborough (2003) used three characteristic groups to explain electricity consumption patterns in the home: “predictable”, “moderately predictable” and “unpredictable”. “Predictable loads” consisted of small cyclic loads occurring when a dwelling is unoccupied or all the occupants are asleep. “Moderately predictable” related to the habitual behaviour of the occupants and “unpredictable” described the vast majority of electricity consumption within a dwelling. The “predictable” component could be classed as a deterministic process, the “unpredictable” component as a stochastic process and the “moderately predictable” somewhere between the two.

An electricity load profile can therefore be described as a combination of deterministic and stochastic processes. For example a cold appliance such as a fridge is usually left on 24 hours a day, would be a deterministic process. This could be approximated as a function of internal

dwelling temperature. The use of other appliances such as kettles are more random and difficult to model and may be a function of various independent variables relating to a dwelling occupant. This introduces a stochastic component to a typical electricity load profile and can be difficult to model.

Markov chain modelling is an autoregressive process that can be used to generate synthetic sequences for modelling stochastic domestic load profiles. This technique has been used in the past to model various applications such as rainfall (Srikanthan, 1985) and wind speed at particular locations (Shamshad et al. 2005). In particular it is suited to modelling systems where the current state of a sequence is highly correlated to the state immediately preceding it and where a large sample size of data exists.

Markov chain modelling is based on the construction of a transitional probability matrix where the transition from one discrete state to another discrete state is represented in terms of its probability. A first order Markov chain model looks at the current state and the one immediately preceding it to calculate the probability of going to the next state. A second order Markov chain model looks at the two previous states and compares with the current state to determine the next state. For a first order Markov chain model, the transitional probability matrix,  $P$ , can be defined with  $p_{k,k}$  probabilities for  $k$  states as follows:

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \dots & p_{1,k} \\ p_{2,1} & p_{2,2} & \dots & p_{2,k} \\ \vdots & \vdots & \vdots & \vdots \\ p_{k,1} & p_{k,2} & \dots & p_{k,k} \end{bmatrix} \quad (1)$$

The state probabilities are calculated by the relative frequencies for each state changing from one to the next. A cumulative probability matrix is calculated by summing the number of frequencies of a particular state,  $n_{i,j}$ , where  $i$  and  $j$  represent different states, and dividing by the total number per state:

$$P_{cum} = \frac{n_{i,j}}{\sum_j n_{i,j}} \quad (2)$$

For each group of states (i.e. each row) the cumulative probability equals one. This represents the relative probability of changing from the current state to every other state including the current state.

A first order Markov chain model using a 24x24 probability matrix was chosen to model individual domestic load profiles based on the distribution of household loads in Ireland. Bin sizes for sampling were chosen based

on standard deviation (0.0837) and mean electricity consumption (0.5525kW) for a sample of 4,500 Irish dwellings. Synthetically generated output values were calculated using a uniformly distributed random number generator choosing a value between each bin width.

The first state of the Markov chain sequence is generated by a random number generator with values between 0 and 1. After the initial state is chosen the transitional probability matrix is used to select every consecutive state after this. The state with the highest probability, which is usually the same state, will be selected most often but will depend upon the probability matrix. This is reflected in the matrix where the largest probabilities are usually located along the diagonal.

Five different dwelling types were modelled by generating transitional probability matrices for detached, semi-detached, bungalow, terraced and apartment dwellings. Six months electricity consumption data, metered at half hourly intervals between 1<sup>st</sup> July 2009 and 31<sup>st</sup> December 2009 was used to calculate the probability matrices.

### 3. Results and Discussion

A Markov chain approach to modelling domestic load profiles was discussed above. A program was coded in Matlab to calculate probability transitional matrices and generate synthetic load profiles for five individual dwelling types based on metered data. Table 1 compares statistical properties between metered and synthetic sequences such as mean, standard deviation (std), maximum and minimum values over a six month period. For each dwelling type the difference between mean and standard deviation for each sequence is less than 6%. However, the synthetic sequence continually over-estimated the mean and standard deviation for each dwelling over the period shown. This is most likely a result of sampling error and could be resolved by further increasing the number of bins at the lower end of a customer load at the expense of higher values. Maximum and minimum values of dwellings load are also shown in the Table 1 below.

Table 2 shows total kWh for each dwelling type for metered and synthetically generated profiles over a one year period. Six months data (July – December 2009) was mirrored to extend to a full years data. This can be compared with national and international benchmarks such as that published by Sustainable Energy Authority of Ireland where it is estimated that an ‘average’ dwelling in Ireland consumed 5,591kWh in 2006 (SEAI, 2008). The error between metered and synthetic profiles is shown with terraced dwelling showing the largest deviation from the real data.

Synthetic sequences were generated for each dwelling type with similar results. Due to space requirements within this paper only figures for a single detached dwelling are shown with results in table form for all dwelling types where appropriate.

Figure 1 shows metered and synthetic sequences over a six month period for a detached dwelling. A simple visual inspection of the sequences indicates that they both compare reasonably well in the time domain and further comparative tests are carried out to determine whether this is the case.

Table 1: Statistical properties for each dwelling type for metered and synthetic profiles (kW).

	<i>Detached</i>	<i>Semi-detached</i>	<i>Bungalow</i>	<i>Terraced</i>	<i>Apartment</i>
Mean (metered)	0.4901	0.5834	0.7460	0.6510	0.1397
Mean (synthetic)	0.5073	0.5917	0.7661	0.6583	0.1436
STD (metered)	0.5969	0.7265	0.7578	0.7023	0.1976
STD (synthetic)	0.6300	0.7477	0.7930	0.7198	0.2021
Max (metered)	5.9820	5.4400	6.6060	5.5980	3.6980
Max (synthetic)	5.7638	5.3840	7.3146	6.4895	3.4000
Min (metered)	0.0800	0	0	0	0
Min (synthetic)	0.0503	0.0002	0.0110	0.0504	0.0001

Table 2: Electricity consumption per dwelling type for metered and synthetic profiles (kWh).

	<i>Detached</i>	<i>Semi-detached</i>	<i>Bungalow</i>	<i>Terraced</i>	<i>Apartment</i>
Metered	4305	5125	6553	5718	1227
Synthetic	4456	5170	6922	6094	1261
Error	3.4%	0.9%	5.3%	6.2%	2.7%

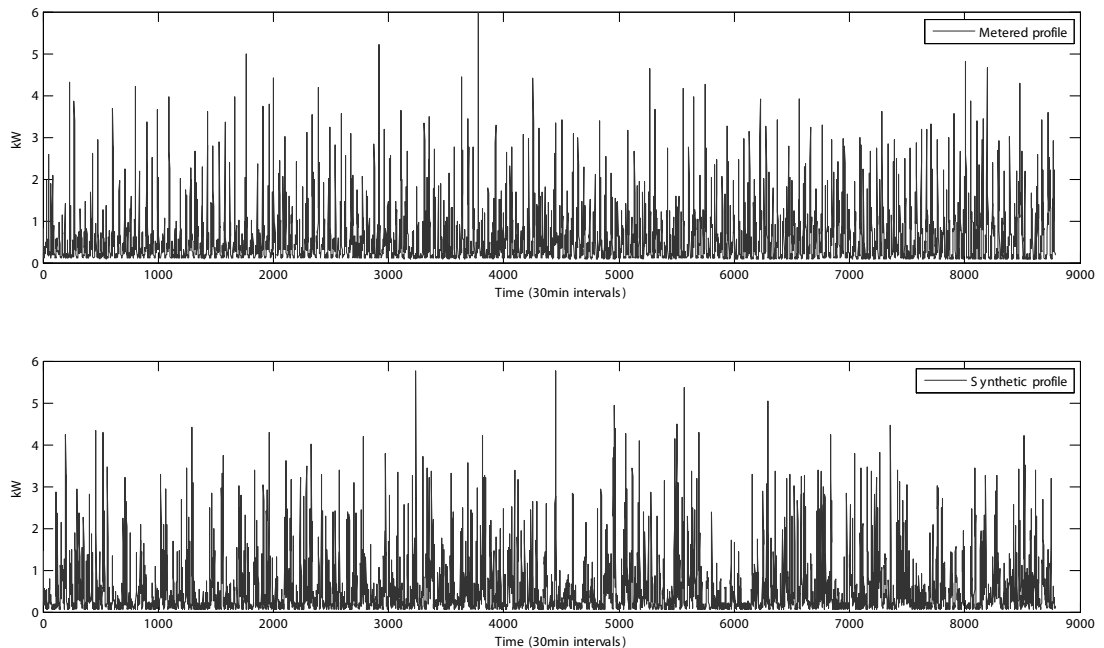


Figure 1 – Detached Dwelling Six Month Profile

Table 3 – Three parameter lognormal for metered and synthetic distribution over 6 month period

	<i>Detached</i>	<i>Semi-detached</i>	<i>Bungalow</i>	<i>Terraced</i>	<i>Apartment</i>
Location (metered)	-1.752	-1.215	-0.7807	-0.8834	-2.316
Location (synthetic)	-1.523	-1.233	-0.7896	-1.003	-2.203
Scale (metered)	1.37	1.162	1.024	0.9482	0.7286
Scale (synthetic)	1.299	1.21	1.073	1.105	0.7696
Threshold (metered)	0.07889	-0.00027	-0.00097	-0.00442	-0.00121
Threshold (synthetic)	0.04463	-0.00056	0.009584	0.03553	-0.012

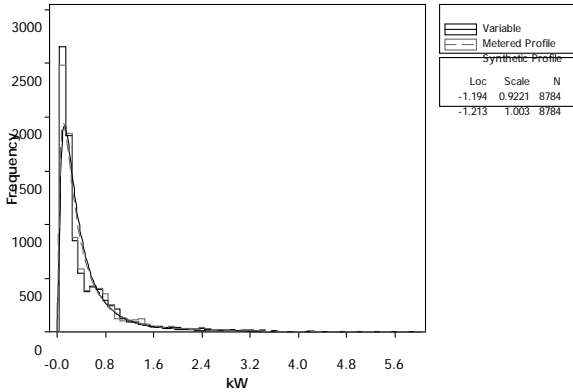


Figure 2 – Detached dwelling histogram for metered and synthetic profiles over six months

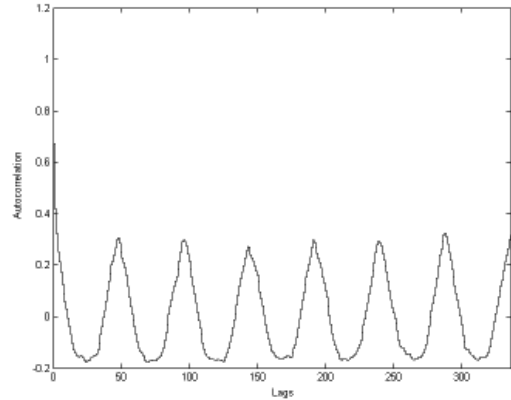


Figure 3 – Autocorrelation function for metered profile of a detached dwelling

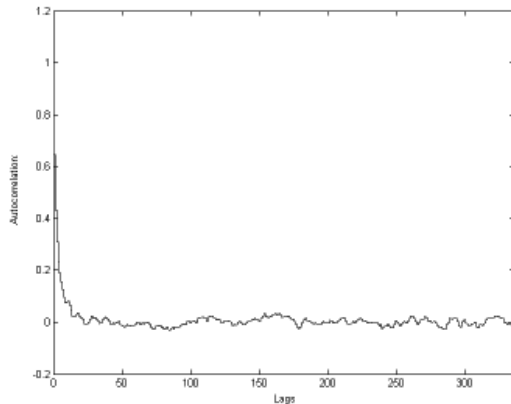


Figure 4 – Autocorrelation function for synthetic profile of a detached dwelling

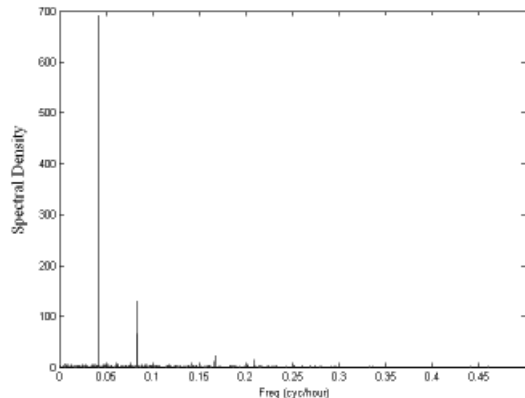


Figure 5 – Spectral periodgram for detached dwelling for metered profile over a six month period

Figure 2 shows the frequency distribution for both sequences. A three parameter log-normal distribution is fitted to the data and location, scale and threshold statistical properties are shown in Table 3. Marginal differences exist between the log normal distribution parameters for metered and synthetic generated sequences.

Figures 3 and 4 show autocorrelation functions for the same detached dwelling for metered and synthetic profiles over a weekly period. A period of one week is shown with lag of half hourly intervals. There is a clear cyclical pattern to the metered data over a 24 hour period showing the high correlation between electricity consumed at the same time interval each day. For the synthetic sequence the autocorrelation function decays to zero almost instantly indicating that the same daily cyclical pattern is not present in the synthetic sequence.

Spectral density functions are also shown for metered and synthetic sequences in Figures 5 and 6 with frequency period in hours. The metered profile shows large frequency components around twelve and twenty-four hour periods which was also reflected in the autocorrelation function. This is in stark contrast to the synthetic sequence where multiple frequency components are shown which don't appear to indicate any clear pattern.

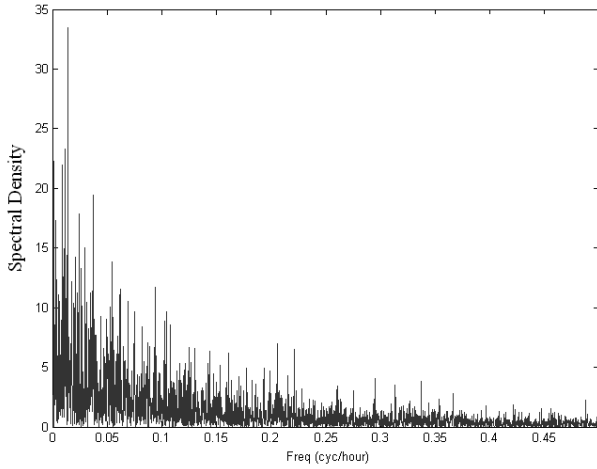


Figure 6 – Spectral periodogram for detached dwelling for synthetic profile over a six month period

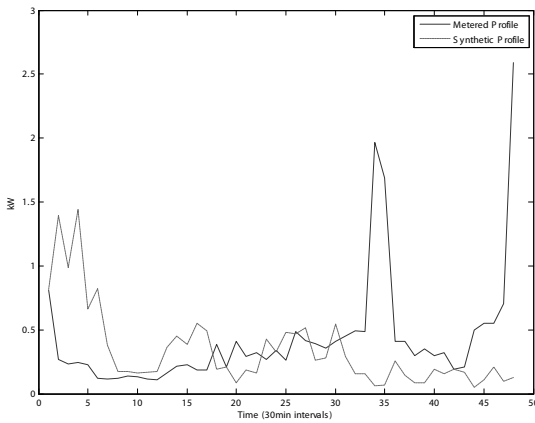


Figure 7 – Detached dwelling daily profile for 01<sup>st</sup> July 2009

Figure 7 shows metered and synthetic sequences for the same detached dwelling on the 1<sup>st</sup> July 2009. It is apparent from Figure 7 that the daily peaks for each profile do not coincide on a time basis. The synthetic profile predicted a daily peak in the early hours of the morning around 1.30am whereas the metered profile showed a daily peak at 5.30pm over a daily period.

The Markov chain process shown above was unable to model the effect of time of day on electricity consumption patterns. This is an obvious flaw to the model where time of day is a major determinant for electricity consumption. Hence daily peaks did not occur at the same time interval. A time component needs to be included as part of the transitional probability matrices.

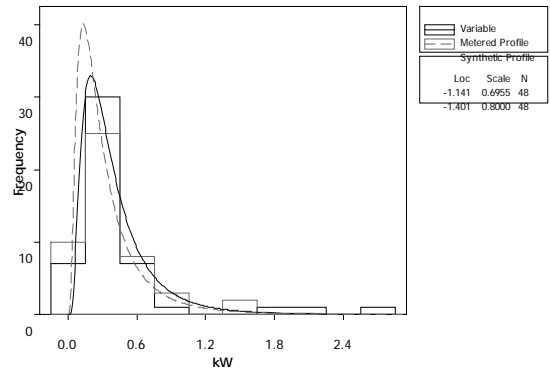


Figure 8 – Detached dwelling daily histogram for 01<sup>st</sup> July 2009

Figure 8 shows the daily distribution of electricity consumption for the detached dwelling. A two parameter log normal distribution is fitted to the data showing location and scale parameters. The synthetic profile slightly underestimated the load in this particular instance with the difference between metered and synthetic generated electricity consumption 10.3kWh compared to 8.2kWh respectively for the 1<sup>st</sup> July 2009. When averaged over time, a clear pattern of a small peak in the morning with a larger peak in the evening and a small baseload over the night time period is apparent. This can be seen in Figure 9 where mean and 95% confidence intervals are shown over a daily period for six months. The synthetic sequence shown in Figure 10 did not reproduce this characteristic profile shape with an almost flat response across the entire day reflecting a mean value for electricity consumption across a random day. It is clear that Figures 9 and 10 represent two distinctly different profiles for the same detached dwelling when compared over the same time intervals.

A large number of independent variables influence the magnitude and time component of electricity consumption. However, time is a major factor in determining the amount of electricity consumed with a dwelling even though the profile may appear to be highly stochastic. In its current form the model is unable to characterise load profiles for individual dwellings as the generated synthetic sequence is independent of time. However, the synthetic sequence generates a good approximation of the total electricity consumed within dwellings as one would expect from an empirical model.

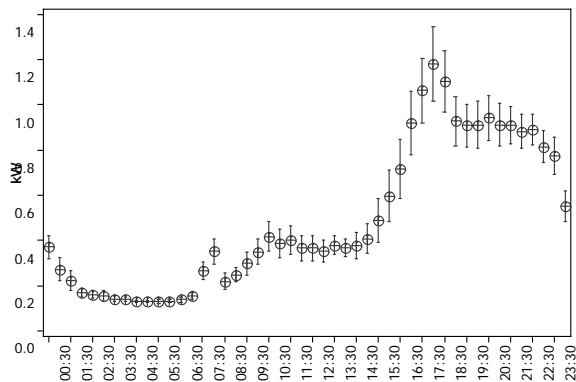


Figure 9 – Mean and 95% Confidence Intervals for detached dwelling over a six month period – metered profile

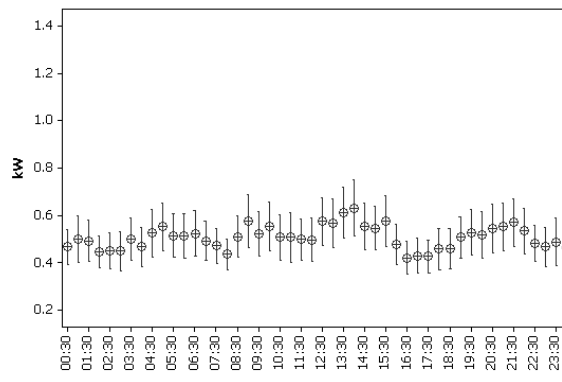


Figure 10 – Mean and 95% Confidence Intervals for detached dwelling over a six month period – synthetic profile

#### 4. Conclusions

A Markov chain model was used to model domestic electricity load profiles using a 24x24 probability matrix. Five different dwelling types were modelled over half hourly intervals and results compared to the original data. Certain key statistical properties such as mean, standard deviation, maximum and minimum values were satisfactory transferred between metered and synthetically generated load profiles. The temporal properties of the synthetic sequence compared poorly with the original data.

The autocorrelation function was not reproduced in the synthetic profile and there was little correlation shown between spectral density plots.

Time of day is a major factor in determining electricity consumption. The Markov chain process was unable to successfully model the time component. The results showed uncharacteristic peak loads occurring at times of the day and night uncommon to typical domestic load profiles.

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# Генерация Отечественных Профилей Нагрузки Электричества через Моделирование Цепи Маркова

Финган J. McLoughlin<sup>1</sup>, Ейдан Duffy<sup>1</sup> and Майкл Conlon<sup>2</sup>

<sup>1</sup> Департамент Строительной Инженерии, Факультет Строительных Услуг, Институт Технологии  
Дублина, ул. Болтон, Дублин 1, Ирландия

<sup>2</sup> Факультет Электротехнических Систем, Институт Технологии Дублина, ул. Кевин, Дублин 8, Ирландия

**Резюме:** Микро-генеративные технологии, такие как фотовольтаики и микро-ветроэнергетика становятся все более популярными среди домовладельцев, в основном, в результате политики механизмов поддержки которые помогают улучшить стоимость конкурентоспособности по сравнению с генерацией традиционным ископаемым топливом. Национальные стратегии правительства сократить спрос на электроэнергию генерирующей из ископаемых топлив и встретить цели 20/20 Европейского Союза ведут эти изменения. Однако реальная эффективность этих технологий в отечественной обстановке, не часто известна, поскольку модели высокого временного анализа для отечественных профилей нагрузки электричества не являются легкодоступными. В результате, прогнозы по снижению спроса на электричество и финансовые окупаемости этих микро-генеративных технологий, не всегда реальные. Отечественные профили нагрузки электричества являются часто очень стохастическими, под влиянием многих различных независимых переменных, таких как относящихся к окружающей среде и жилых характеристик, которые формируют нагрузки индивидуального клиента через один день. Эта статья представляет собой стохастический метод для профилей нагрузки генерации электричества, основанный на применении процесса цепи Маркова. Потребление электроэнергии было зафиксировано на полу-часовых интервалов в течение шести месяцев для пяти индивидуальных ирландских типов жилья и генерировало профили нагрузки синтетического электричества. Целью данной работы является определение того, является ли моделирование цепи Маркова эффективным способом регенерировать профилей нагрузки электричества для отечественных жилищ и выяснение недостатков с помощью этой особенной техники. Результаты показывают, что величина компонента профиля нагрузки может быть воспроизведена эффективно а временное распределение необходимо адресовать дальше.

**Ключевые слова:** Цепь Маркова, электричество